MODELLING DYNAMIC BEHAVIOUR OF WATER DISTRIBUTION SYSTEMS FOR CONTROL PURPOSES

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ABSTRACT

Water distribution systems (WDSs) are getting equipped with advanced feedback loops, which require specialised methods and software tools to model such systems. The aim of this work is to demonstrate the usefulness of the rigid water column (RWC) model to analyze the dynamic interactions between these loops and system stability. Our work fills the gap between extended period simulation, which is used for steady state analysis, and transient simulation, which is used for surge analysis. The paper proposes a generic dynamic WDS model where pipes are represented by the RWC model, while control valves, pumps and tanks are represented by algebraic or ordinary differential equations. The model has been implemented in the MATLAB/Simulink environment, which provides a rich library of control components and algorithms for implemention of complex mathematical models. An industrial case study which prompted the development of the methodology is also presented.

INTRODUCTION

Water distribution systems (WDSs) should be equipped with monitoring and control instrumentation to secure water supply to users, minimize water losses and energy consumption, provide effective maintenance and handle contingency situation, (Allen et al. 2012), (Cabrera 2003), (Giudicianni et al. 2020). Such instrumentation includes, remotely controlled valves, (Creaco and Walski 2018), application of pumps as turbines (PAT) for pressure control and energy recovery, (Fontana et al. 2016), and district metering areas with dynamic boundaries, (Wright et al. 2014). An up to date review of the applications of the advanced control elements is presented in (Creaco et al. 2019). This can lead to a multi-layer control system where the bottom layer is directly connected to the water system by collecting measurements and actuating control elements responsible for the dynamic behaviour of the system.

The dynamics are caused by water inertia and water compressibility, as well as the dynamics of the control elements. For instance, a hydraulically control pressure reducing valve (PRV) can take from tens of seconds to a minute to settle to a new position. Similarly, a variable speed pump, has the time constant of tens of seconds due to combined inertia of a motor and a pump. WDSs as such are stable, (Masuda and Meng 2019) and transient behaviour caused by switching events in the system decay relatively quickly to zero (in a matter of seconds or minutes). However, introducing feedback loops may change the situation.

While they are useful if properly designed in terms of stability and robustness (Janus and Ulanicki 2018), (Galuppini et al. 2020), they can otherwise lead to instability and substantial material losses, (Ulanicki and Skworcow 2014). The use of models to design and operate WDSs is widely accepted, including by regulatory bodies such as OFWAT in the UK. From the physics point of view, models can be categorised with respect to physical accuracy into 1) transient models where pipes are described by partial differential equations, 2) rigid water column (RWC) models where pipes and control elements are described by ordinary differential equations (ODEs) and 3) extended period simulation models were pipes and control elements are described by algebraic equations and tanks by differential equations, (Walski et al. 2003). There are software tools for transient modelling, for instance (Bentley 2020) that are used mainly for surge analysis and events such as ‘pump trips’. Extended period simulation is now used across the world thanks to availability of the open source EPANET software (Rossman et al. 2000). There is no widely available software for RWC modelling although a significant progress was recently achieved in the development of efficient numerical algorithms for solving such models, (Nault and Karney 2016b) and (Nault and Karney 2016a). These papers provide also an excellent review of the recent work in this area.

In (Nault and Karney 2016b), the authors proposed an RWC global gradient algorithm (GGA) which is a generalisation of the well-known GGA, (Todini and Pilati 1988) used by EPANET 2 for EPS simulations. An implicit method of integration of the ordinary differential equations resulting from the RWC model ensured numerical stability of the calculation scheme. Efficiency of the algorithm was further improved in (Nault and Karney 2016a) by introducing a mechanism to decide whether a given pipe should be modeled as a dynamic or static component. The RWC approach finds applications not only in WDS analysis but also in other engineering areas, for instance for analyzing the emptying process in a pipeline using pressurized air, (Coronado-Hernández et al. 2018) or for analysis of slow oscillations between the lake and the surge tank in a hydro-power station, (Zhang 2020).

This paper proposes an alternative approach to that presented in (Nault and Karney 2016b) by using an existing general purpose software environment and is geared towards analysis of the dynamics of WDSs equipped with many advanced control elements. The implementation in MATLAB/Simulink proposed in this work creates an opportunity to use a readily available simulation engine as well as tools for identification, control and optimisation. In particular, MATLAB allows user defined control algorithms to be included as part of a Simulink model.A typical application of the methodology is for normal operating conditions over extended periods of time in a day or a week. To investigate abrupt changes like complete valve closures, it is recommended to use a transient model.

The paper has three main sections: Modelling Principles, Dynamic WDS Model and Case Study. The first part of the Modelling Principles section introduces individual components of the model, which include pipe, TCV, PRV, pump, tank and leakage. A generic form of a dynamic (described by ODEs) control element is proposed as well. The second part of that section explains connection equations resulting from the mass balance at nodes and energy balance in loops. In the Dynamic WDS Model section, equations are put together to represent a generic dynamic WDS model. The section also shows how to implement the model in the MATLAB/Simulink environment, (MathWorks 2020). The theory was inspired by industrial case studies one of which is presented in the Case Study section. The results have supported the design and operational decisions of the industrial partner.

MODELLING PRINCIPLES

Modelling principles are similar to those used to derive an extended period simulation model. Three groups of equations are required: mass balance at nodes, energy balance in loops and components equations. One difference is that now the components can be described by ordinary differential equations which require special treatment.

Individual Component Equations

Pipe

A pipe is represented by a standard rigid column model which accounts for the inertia of the water column but ignores water compressibility. The model is given by the equation

$$\dot{q} = -Rq|q| + M(h_o - h_d) \tag{1}$$

where $R = f_D \frac{1}{2DA}$, $M = \frac{gA}{L}$, f_D is the Darcy friction factor, D is the pipe hydraulic diameter, A is the pipe cross section area, L is the pipe length and g is the gravitational acceleration.

Throttle Control Valve (TCV)

A TCV is a static component normally used to increase or decrease flows or to control pressures in a water system. TCVs are modelled by the equation

$$h_o - h_d = K_v \frac{1}{2gA_v^2} q^2 \tag{2}$$

where h_o is the head at the origin node, h_d is the head at the destination node, q is the valve flow, K_v is a minor loss coefficient, which depends on the valve opening, and A_v is the cross-section area of the valve.

Pressure Reducing Valve (PRV)

A PRV is a two-port one-directional component. A PRV connects two nodes, the origin node, h_o and the destination node h_d. The role of a PRV is to maintain the outlet head of the valve (set-point) constant independently of the inlet head, provided that the inlet head is higher than the set-point. A hydraulically controlled PRV is a dynamic element with a local feedback loop via a pilot valve. When the set-point is changed it takes some time before the output, h_d, settles to a new value.

The dynamic part of the model with the set-point, h_{set}, is given by Eqs 3, (Prescott and Ulanicki 2003),

$$\begin{aligned} \dot{x}_m &= \alpha_{open}(h_{set} - h_d) \\ \dot{x}_m &= \alpha_{close}(h_{set} - h_d) \end{aligned} \tag{3}$$

where x_m is the valve opening in percentage and α_{open} and α_{close} are the rates of the valve opening and closing, respectively. The algebraic part is given by a standard valve equation $q = C_v(x_m)\sqrt{h_o - x_d}$ which can be converted to Eqs 4,

$$\begin{aligned} h_o - h_d =& \frac{1}{C_v(x_m)^2} q^2 \\ C_v(x_m) =& A_{cv} x_m^2 + B_{cv} x_m + C_{cv} \end{aligned} \tag{4}$$

where $C_v(x_m)$ is the valve capacity which depends on the valve opening x_m. This relationship is usually provided by a valve manufacturer and can be accurately approximated by a second order polynomial as indicated in Eqs 4.

Pump

A pump provides energy to overcome gravity and friction energy losses in pipes. The pump is represented by two equations. The first equation (Eq. 5)

$$\dot{s} = -\frac{1}{T}s + \frac{1}{T}v \tag{5}$$

is a differential equation that describes the pump inertia, where s is the pump speed, v is the speed set-point, and T is a time constant. The second is an algebraic equation that describes the head increase across the pump. It has the form (Ulanicki et al. 2008), (Walski and Barnard 2004),

$$h_d - h_o = A\left(\frac{q}{u}\right)^2 + B\left(\frac{q}{u}\right)s + Cs^2 \tag{6}$$

scaled by speed s and number u of pumps connected in parallel.

Storage reservoir (Tank)

A storage reservoir (tank) can have a separate inlet and outlet, e.g., for a top-fed tank, or have one feed in which the flow can change direction. The same equation (Eq. 7) is used for both types. The rate of change of the tank head $\dot{h}_t$ depends on the balance between the inflow q_{in} and the outflow q_{out}.

$$\dot{h}_t = \frac{1}{A_t}q_{in} - \frac{1}{A_t}q_{out} \tag{7}$$

where A_t is the cross-section area of the tank. The flows q_{in} and q_{out} can be associated with a valve or a pump for pumped tanks or with a network node for a floating tank. In the latter case, it is only one flow, q_t,which changes direction.

Fixed Head Reservoir

This is a component to represent a significant reservoir with a constant head independent of inflows/outflows associated with the reservoir. It is represented by the equation

$$h_0 = constant \tag{8}$$

Other components

Other components can be added to the list as required. They can be static like TCV or dynamic like PRV. A generic dynamic component has a static part, Eq. 9, and a dynamic part given by Eq. 10,

$$h_o - h_d = g(q, a, c) \tag{9}$$

$$\dot{a} = f(a, c, h_o, h_d, q) \tag{10}$$

where a is a state variable of the component, e.g., valve opening, x_m for the PRV or the pump speed s, for the pump, and c is a control variable such as a PRV or a pump set-point. The shape of functions g and f depends on the nature of the individual component.

A useful static component is leakage described by the standard orifice equation $q_l = A_l\sqrt{2g(h_l - h_{elev})}$ which can be converted to Eq. 11

$$h_l - h_{elev} = \frac{1}{2gA_l^2} q_l^2 \tag{11}$$

where h_l is the head at the leakage node, h_{elev} is the elevation of the node, A_l is the leakage area and q_l is the leakage flow.

Equations of Connections

Individual components are connected into a functional network which represents a physical WDS. The network topology is encoded into a node-element $n \times e$ incidence matrix, $\mathbf{\Lambda}$ (Bryds and Ulanicki 1994) where the n rows correspond to the nodes and the e columns correspond to the elements (components). A nonzero entry $\lambda_{i,j}$ of the matrix $\mathbf{\Lambda}$ indicates that element j is connected to node i; it assumes value $+1$ if the element enters the node and -1 if the element leaves the node. Nodes and elements can be divided into different categories. The set of all nodes $\mathbb{N}$ is the union of the set of connection nodes $\mathbb{N}_c$ and the set of nodes $\mathbb{N}_f$ with forced head, such as fixed grade nodes or tank nodes, That is,

$$\mathbb{N} = \mathbb{N}_c \cup \mathbb{N}_f, \quad n = n_c + n_f \tag{12}$$

where n_c and n_f are the number of connection nodes and forced head nodes, respectively. The forced head nodes can be divided further into source nodes (tanks, reservoirs) and sink nodes such as leakage nodes or feed nodes for the top fed tanks. The set of connection nodes is split further into the two disjoint sets, $\mathbb{N}_{c,p}$ for nodes connected to pipes only and $\mathbb{N}_{c,ctr}$ for nodes with pipes and control elements,

$$\mathbb{N}_c = \mathbb{N}_{c,p} \cup \mathbb{N}_{c,ctr}, \quad n_c = n_{c,p} + n_{c,ctr} \tag{13}$$

where $n_{c,p}$ and $n_{c,ctr}$ are the number of nodes in the sets $\mathbb{N}_{c,p}$ and $\mathbb{N}_{c,ctr}$, respectively.

The set of all elements, $\mathbb{E}$ is the union of the set of pipes, $\mathbb{E}_p$, and the set of control elements, $\mathbb{E}_{ctr}$,

$$\mathbb{E} = \mathbb{E}_p \cup \mathbb{E}_{ctr}, \quad e = e_p + e_{ctr} \tag{14}$$

where e_p is the number of pipes and e_{ctr} is the number of control elements. It is useful to divide the set $\mathbb{E}_{ctr}$ of control elements into two sets, the set $\mathbb{E}_{ctr,c}$ of the control elements connected between two connection nodes and the set $\mathbb{E}_{ctr,f}$ of control elements with one node connected to a forced head node,

$$\mathbb{E}_{ctr} = \mathbb{E}_{ctr,c} \cup \mathbb{E}_{ctr,f}, \quad e_{ctr} = e_{ctr,c} + e_{ctr,f} \tag{15}$$

where $e_{ctr,c}$ is the number of control elements connected between two connection nodes and $e_{ctr,f}$ is the number of control element with one node connected to a forced head node.

There are two physical laws governing WDS models, mass balance at connection nodes and energy balance in loops. Mass balance for connection nodes is given by Eq. 16, (Todini and Pilati 1988), (Bryds and Ulanicki 1994),

$$\mathbf{\Lambda}_c \times \mathbf{q} = \mathbf{d}_c \tag{16}$$

where $\mathbf{\Lambda}_c$ is the part of matrix $\mathbf{\Lambda}$ with n_c rows corresponding to the connection nodes, i.e.,

$$\mathbf{\Lambda} = \begin{bmatrix} \mathbf{\Lambda}_c \\ \mathbf{\Lambda}_f \end{bmatrix}$$

and $\mathbf{\Lambda}_f$ is the part which corresponds to the forced head nodes, $\mathbf{q}$ is the vector of e flows in all elements (pipes and control elements) and $\mathbf{d}_c$ is the vector of nodal demands at connection nodes. In a WDS model there are $(e - n + 1)$ fundamental loops created by adding elements to a spanning tree of the WDS model. There are also $(n_f - 1)$ pseudo-loops which are paths leading from a forced head node selected as the reference node and the remaining forced head nodes.The energy conservation law is formulated with the help of the loop-element incidence matrix Γ.

$$\mathbf{\Gamma} = \begin{bmatrix} \mathbf{\Gamma}_l \\ \mathbf{\Gamma}_{f-1} \end{bmatrix} \tag{17}$$

It has a row for each loop and a column for each element. For a given loop, the matrix entry is + 1 if the orientation of an element in the loop is consistent with the orientation of the loop, and -1 for the opposite orientation. If the element is not in the loop, the entry is 0. The block $\mathbf{\Gamma}_l$ is responsible for the fundamental loops and $\mathbf{\Gamma}_{f-1}$ is responsible for the pseudo-loops. The energy conservation law takes the form given in Eq 18,

$$\mathbf{\Gamma} \cdot \Delta\mathbf{h} = \begin{bmatrix} \mathbf{0} \\ -\Delta\mathbf{h}_{f-1} \end{bmatrix} \tag{18}$$

where $\Delta\mathbf{h}$ is the vector of head losses (gains) across all elements and $\mathbf{\Delta}h_{f-1}$ is the vector of the head differences between the reference node and other forced head nodes.

The mass balance and energy balance equations need to be combined with the pipe and other element equations. It is possible to choose arbitrary $(e - n_c)$ independent pipe flows described by the differential equation, Eq. 19,

$$\dot{q}_j = -R_j q_j |q_j| + M_j \Delta h_j, \; j \in E_{p,ind} \tag{19}$$

where $E_{p,ind}$ is a set of independent pipe flows with $(e - n_c)$ elements and Δh_j is the head loss across pipe j. The remaining flows can be calculated from the node mass balance (Eq. 20). In order to prepare mass balance equations for the remaining dependent flows, let us partition $\mathbf{\Lambda}_c$ as $\mathbf{\Lambda}_c = \begin{bmatrix} \mathbf{\Lambda}_{c,dep} & \mathbf{\Lambda}_{c,p,ind} \end{bmatrix}$, where $\mathbf{\Lambda}_{c,dep}$ corresponds to the dependent flows in the pipes and control elements and $\mathbf{\Lambda}_{c,p,ind}$ corresponds to the selected independent pipe flows. This gives

$$\mathbf{\Lambda}_{c,dep} \cdot \mathbf{q}_{dep} = -\mathbf{\Lambda}_{c,p,ind} \cdot \mathbf{q}_{p,ind} + \mathbf{d}_c \tag{20}$$

$\mathbf{\Lambda}_{c,dep}$ is an $n_c \times n_c$ non-singular matrix, hence the vector of dependent flows, $\mathbf{q}_{dep}$ with n_c elements can be calculated by solving the linear algebraic equation, Eq. 20.

The number of unknown variables in the whole WDS model is equal to $(e + n_c)$, i.e., flows in all components, vector $\mathbf{q}$ and heads at connection nodes, vector $\mathbf{h}_c$. The independent pipe flows are described by the differential RWC Eq.19. In order to make sure that the dependent pipe flows satisfy the same RWC equation and also that the number of the unknown variables and the number of equations are the same in the final model it is necessary to introduce additional 'differential' mass balance equations at the connection nodes.

Consider the ith, $i \in \mathbb{N}_{c,p}$, mass balance equation selected from Eq. 16 and differentiate both sides of this equation to obtain Eq. 21,

$$\sum_{j \in \mathbb{E}_i} \lambda_{i,j} \dot{q}_j = \dot{d}_i \tag{21}$$

where $\mathbb{E}_i$ is the set of pipes connected to node i. This set corresponds to the nonzero elements in row i of matrix Λ_c. By replacing $\dot{q}_j$ with the right-hand side of Eq. 19, the following 'differential' mass balance equation for the pipes only nodes is obtained.

$$\sum_{j \in \mathbb{E}_i} \lambda_{i,j} (-R_j q_j |q_j| + M_j \Delta h_j) = \dot{d}_i, \quad i \in \mathbb{N}_{c,p} \tag{22}$$

Let us focus now on mass balance for the connection nodes with a control element. Consider the jth control element with the destination node indexed by $i_{d,j}$ and the origin node indexed by $i_{o,j}$. Mapping between sets of indices can be obtained from matrix Λ_c.

If the mass balance for the destination node and the origin node are added together, the control component flow is eliminated and the result is displayed in Eq. 23,

$$\sum_{m \in \mathbb{E}_{i_{d,j}}} \lambda_{i_{d,j},m}(-R_m q_m|q_m| + M_m \Delta h_m) +$$
$$+ \sum_{m \in \mathbb{E}_{i_{o,j}}} \lambda_{i_{o,j},m}(-R_m q_m|q_m| + M_m \Delta h_m) = \dot{d}_{i_{d,j}} + \dot{d}_{i_{o,j}} \quad j \in \mathbb{E}_{ctr,c} \tag{23}$$

where $\mathbb{E}_{i_{d,j}}$ is a set of pipes connected to the destination node of the control element j, and $\mathbb{E}_{i_{o,j}}$ is a set of pipes connected to the origin node of the control element. If one node of a control component is connected to a forced head node, it is not necessary to consider the mass balance Eq. 23.

DYNAMIC WDS MODEL

Equations from the Modelling Principles section are selected and organised into a system of differential algebraic equations (DAEs). A general semi-explicit DAE system can be written as,

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{y}, \mathbf{u}) \tag{24}$$

$$\mathbf{g}(\mathbf{x}, \mathbf{y}, \mathbf{u}) = \mathbf{0} \tag{25}$$

Eq. 24 is called the differential part and Eq. 25 is called the algebraic part, $\mathbf{x}$ is the state vector, $\mathbf{y}$ is the vector of algebraic variables and $\mathbf{u}$ is the vector of control variables. If the algebraic Eq. 25 can be solved with respect to $\mathbf{y}$ for given $\mathbf{x}$ and $\mathbf{u}$, then, the system has index 1 and is relatively easy to solve numerically.There are two principal approaches to solve numerically the DAE equations. In the first approach, at each iteration of integrating Eq. 24 the algebraic Eq. 25 is solved with respect to $\mathbf{y}$ (adopted in this paper). In the second approach, the algebraic part, Eq. 25 is converted into a differential equation whose solution converges to the solution of the original algebraic equation, (Goman 1986).

Differential part

The differential part is assembled from Eq. 19, Eq, 10 and Eq. 7,

$$\dot{q}_j = -R_j q_j |q_j| + M_j \Delta h_j,\ \ j \in \mathbb{E}_{p,ind} \tag{26}$$

$$\dot{a}_j = f_j(a_j, c_j, h_{o,j}, h_{d,j}, q_j) \quad j \in \mathbb{E}_{c,dyn} \tag{27}$$

$$\dot{h}_{t,j} = \frac{1}{A_{t,j}} q_{in,j} - \frac{1}{A_{t,j}} q_{out,j} \quad j \in \mathbb{T} \tag{28}$$

Eq. 26 represents independent pipe flows,while Eq. 27 represents the dynamic part of the control elements such as PRVs and pumps which belong to $\mathbb{E}_{c,dyn}$ - the set of the dynamic control elements. Note that $\mathbb{E}_{c,dyn}$ is a subset of the set, $\mathbb{E}_c = \mathbb{E}_{c,dyn} \cup \mathbb{E}_{c,stat}$. Eq. 28 is for tanks and is evaluated depending on the type of the tank. For the pumped tank, $q_{in,j}$ will be a flow in the element feeding the tank and for the floating tank it will result from the mass balance at the tank node.

Algebraic Part

The algebraic part is assembled from Eq. 20, Eq. 9, Eq. 22 and Eq. 23,

$$\mathbf{\Lambda}_{c,dep} \cdot \mathbf{q}_{dep} = -\mathbf{\Lambda}_{c,p,ind} \cdot \mathbf{q}_{p,ind} + \mathbf{d}_c \tag{29}$$

$$\Delta h_j = g_j(q_j, a_j, c_j), \quad j \in \mathbb{E}_{ctr,f} \tag{30}$$

$$\Delta h_j = g_j(q_j, a_j, c_j), \quad j \in \mathbb{E}_{ctr,c} \tag{31}$$

$$\sum_{j \in \mathbb{E}_i} \lambda_{i,j} M_j \Delta h_j = \sum_{j \in \mathbb{E}_i} \lambda_{i,j} R_j q_j |q_j| + \dot{d}_i,\ \ i \in \mathbb{N}_{c,p} \tag{32}$$

$$\begin{gathered}\sum_{m \in \mathbb{E}_{i_{d,j}}} \lambda_{i_{d,j},m} M_m \Delta h_m + \sum_{m \in \mathbb{E}_{i_{o,j}}} \lambda_{i_{o,j},m} M_m \Delta h_m = \\ \sum_{m \in \mathbb{E}_{i_{o,j}}} \lambda_{i_{o,j},m} R_m q_m |q_m| + \sum_{m \in \mathbb{E}_{i_{d,j}}} \lambda_{i_{d,j},m} R_m q_m |q_m| + \dot{d}_{i_{d,j}} + \dot{d}_{i_{o,j}}, \quad j \in \mathbb{E}_{ctr,c}\end{gathered} \tag{33}$$

$$\mathbf{\Gamma}_p \times \Delta \mathbf{h}_p = -\mathbf{\Gamma}_{ctr} \times \Delta \mathbf{h}_{ctr} + \begin{bmatrix} \mathbf{0} \\ -\Delta \mathbf{h}_{f-1} \end{bmatrix} \tag{34}$$

where the sets $\mathbb{E}_{ctr,f}$, $\mathbb{E}_{ctr,c}$ and $\mathbb{N}_{cp}$ are defined by Eq. 15 and Eq. 13, respectively. The unknown variable is the vector of head losses across all elements, $\Delta\mathbf{h}$. The mass balance Eq. 29 allows to calculate the vector of dependent flows $\mathbf{q}_{dep}$ for given independent pipe flows, $\mathbf{q}_{p,ind}$. Eq. 30 and Eq. 31 give directly head losses across all control elements.

Eq. 32, Eq. 33 and Eq. 34 define e_p simultaneous linear equation, with e_p unknown head losses across all pipes. Notice that the unknown variables in these equations form the vector of head losses on pipes only, $\Delta\mathbf{h}_p$, as the vector of head losses on control elements, $\Delta\mathbf{h}_c$, is already known from Eq. 30 and Eq. 31. The loop-element incidence matrix $\mathbf{\Gamma}$ has been partitioned into two parts: one corresponding to pipes, $\mathbf{\Gamma}_p$, and another corresponding to control elements, $\mathbf{\Gamma}_{ctr}$.

The $n_{c,p}$ equations in Eq. 32, $e_{ctr,c}$ equations in Eq. 33, and $(e - n_c)$ energy balance equations in Eq. 34 form $(n_{c,p} + e_{ctr,c} + e - n_c)$ simultaneous equations, which is the number of pipes e_p in the model as explained next. Since, $(n_c - n_{c,p} = 2e_{ctr,c} + e_{ctr,f})$ the number of simultaneous equations is thus

$$\begin{aligned} no\_of\_eqs &= (e - 2e_{ctr,c} - e_{ctr,f} + e_{ctr,c}) = \\ &= (e - (e_{ctr,c} + e_{ctr,f}) = (e - e_{ctr}) = e_p \end{aligned} \tag{35}$$

The solution is obtained in terms of head losses across elements. Subsequently, the values of the heads at the junction nodes can be obtained one by one, starting calculations from the forced head nodes or by solving the linear equation $(\mathbf{\Lambda}_c\mathbf{\Lambda}_c^T)\mathbf{h}_c = \mathbf{\Lambda}_c\Delta\mathbf{h} + \mathbf{\Lambda}_c\mathbf{\Lambda}_f^T\mathbf{h}_f$. After a relatively lengthy derivation, which is omitted here, it is possible to express Eq. 32 and Eq. 33 in the matrix-vector form,

$$\mathbf{\Lambda}_{c,red} \times \mathbf{M} \times \Delta\mathbf{h}_p = \mathbf{\Lambda}_{c,red}\mathbf{R}(\mathbf{q}) + \mathbf{d}_{c.red} \tag{36}$$

where $\mathbf{\Lambda}_{c,red}$ is a reduced node-element incidence matrix with $(n_{c,p} + e_{ctr,c})$ rows and e_p columns corresponding to a reduced network from which the control elements have been removed and their respective nodes merged, $\mathbf{M}$ is an $(e_p \times e_p)$ diagonal matrix whose diagonal entries $M_{j,j}$are equal to M_j , $\mathbf{R}(\mathbf{q})$ is the column vector whose components are $R_j q_j |q_j|$ and finally $\mathbf{d}_{c.red}$ is a reduced and aggregated demand vector.

It was assumed so far that only one control element can be connected to a node. This is consistent with real-world applications and is an assumption made in EPANET as well. Nevertheless, the proposed methodology is able to handle exceptions where many control elements may be connected to a node. In such situations, it is necessary to merge all nodes of the control elements into one and to create a 'differential' mass balance equations (Eq. 33) for this node. One such example is shown in the case-study.

Let us map the state and algebraic variables to the general notation introduced in Eq. 24 and Eq. 25.

$$\mathbf{x} = \begin{bmatrix} \mathbf{q}_{p,ind} \\ \mathbf{a} \\ \mathbf{h}_t \end{bmatrix} \quad \mathbf{y} = \begin{bmatrix} \mathbf{q}_{dep} \\ \Delta\mathbf{h} \end{bmatrix} \tag{37}$$

For a given state vector $\mathbf{x}$, it is possible to solve the algebraic part of the model (Eq. 29 - Eq. 33) and obtain $\mathbf{q}_{dep}$ and $\Delta\mathbf{h}$. The structure of the dynamic WDS model implemented in MATLAB/Simulink is shown as a block diagram in Fig. 1. The graphical representation is directly copied from SIMULINK. Rectangles (blocks) represent causal input-output models where inputs are causes and the outputs are effects. Outputs from one block can be connected to inputs of another block creating a complex block diagram. The outputs at each time step are calculated from inputs by internal algorithms encoded inside the blocks. The time progression is driven by the SIMULINK integration engine, where the typical time horizon is 1 day or 1 week to investigate the behaviour of the system over the entire consumption pattern

There are two major blocks, 'a differential part' which calculates the right-hand sides of Eq. 26 - Eq. 28 and 'an algebraic part'. which solves Eq. 29 -Eq. 34. The outputs from the differential part are derivatives of the state vector which includes a vector of derivatives of independent pipe flows (denoted as qpind_dot in Fig. 1), vector of derivatives of the dynamic variables of the control elements (a_dot) and the vector of derivatives of the tank levels (ht_dot). The blocks with transfer function $\frac{1}{s}$ are integrators to obtain the state vector.The state vector is fed back to the input of the block in order to calculate the right-hand side of the differential equations, Eq. 26, Eq. 27 and Eq. 28. Additional input signals required by this block are: the vector of heads for the fixed grade nodes, $\mathbf{h}_0$, the vector of control variables $\mathbf{c}$ such as pump speed set-points or minor loss coefficients for TCVs, the vector of algebraic variables coming from the algebraic part and finally the vector of demands associated with tanks. The input signals to the algebraic part are the state vector coming from the differential part and the vector of nodal demands. The vector of dependent flows, $\mathbf{q}_{dep}$ and the vector of heads at the connection nodes, $\mathbf{h}_c$, are outputs from the algebraic part. It is standard practice to prepare all data required by a Simulink model in a special initialization script written in the MATLAB language. In the following, we give the pseudocodes of the initialization script, differential part, and algebraic part.

Data initialisation pseudocode

- Initialize nodal sets: $\mathbb{N}$, $\mathbb{N}_c$, $\mathbb{N}_f$, $\mathbb{N}_{c,p}$
- initialize element sets: $\mathbb{E}$, $\mathbb{E}_p$, $\mathbb{E}_{ctr}$, $\mathbb{E}_{ctr,c}$, $\mathbb{E}_{ctr,f}$
- initialize nodal matrices; $\mathbf{\Lambda}$, $\mathbf{\Lambda}_c$, $\mathbf{\Lambda}_f$, $\mathbf{\Lambda}_{c,dep}$, $\mathbf{\Lambda}_{c,p,ind}$, $\mathbf{\Lambda}_{c,red}$
- initialize loop matrices: $\mathbf{\Gamma}$, $\mathbf{\Gamma}_l$, $\mathbf{\Gamma}_{f-1}$, $\mathbf{\Gamma}_p$, $\mathbf{\Gamma}_{ctr}$
- initialize parameters of pipes
- initialize parameters of control elements
- initialize demands
- define initial conditions for the state vector $\mathbf{x}(0)$

Differential part pseudocode

- calculate the right-hand side of Eq. 26
- calculate the right-hand side of Eq. 27
- calculate the right-hand side of Eq. 28

Algebraic part pseudocode

- calculate $\mathbf{q}_{dep}$ by solving the linear algebraic Eq. 29
- calculate the head losses on control elements $\Delta\mathbf{h}_{ctr}$, from Eq. 31 and Eq. 30 by substituting the known element flows to these equations
- calculate the head loss across pipes, $\Delta\mathbf{h}_p$, by solving the system of simultaneous linear equations consisting of Eq. 32, Eq. 33 and Eq. 34
- Calculate vector $\mathbf{h}_c$ from $\Delta\mathbf{h}$ and $\mathbf{h}_f$. Vector $\mathbf{h}_c$ can be calculated by a simple software routine or by solving the following linear equation $(\mathbf{\Lambda}_c\mathbf{\Lambda}_c^T)\mathbf{h}_c = \mathbf{\Lambda}_c\Delta\mathbf{h} + \mathbf{\Lambda}_c\mathbf{\Lambda}_f^T\mathbf{h}_f$

In Fig. 2, the WDS dynamic model is encapsulated in one block called Water Distribution System and external feedback loops are included in the Controllers block. The input signals to this block are state and algebraic variables of the WDS dynamic model.The output is vector **c** of the control variables which may include, for instance, a minor losses coefficient for a valve, a pump speed set-point, a pump control, and a PRV set-point.

Many control functions available in MATLAB/Simulink facilitate implementation of the controllers. The most general facility is the MATLAB Function block which enables to code a user-defined algorithm in the MATLAB programming language. Examples of such algorithms are given in the Case Study section.

CASE STUDY

The proposed approach was applied in a feasibility study to a strategic water supply system which is under development in Belgium. Some elements of the system are already constructed while others including design and control options are under investigation. The aim of this work was to evaluate the dynamic behaviour of the system under different scenarios. A schematic of the part under consideration is displayed in Fig. 3

The source of water is a big reservoir with a head of $322.5m$. A $46km$ long pipe with a diameter of 800mm conveys the water to a PRV chamber. The role of the PRV is to maintain an outlet head of $254m$ from an inlet head of approximately $321m$. The valve chamber can also accommodate a TCV valve whose role will be explained later. Water is transported further in a $9km$ pipe to a connection with a pump station represented by head h_5 in Fig. 3. There is a constant demand d_1 of $5000m^3/d$, which represents an export of water to another water company. Node h_5 is a mixing node where water from the main pipeline is being mixed with water pumped from a free surface tunnel. The mixed water is transported to a top fed tank of capacity $1000m^3$, with the inflow controlled by TCV2. The downstream part of the system is represented by demand d_3. This is a major demand with significant variability over a day. The tank outflow, d_2, covers the local demand with a typical flow pattern and average value of $2500m^3/d$. Pipe data are shown in Table 1, and the control elements data are shown in Table 2.

The water in the main pipe has a different quality from the water pumped from the tunnel. The main objective was to investigate the feasibility of maintaining a constant ratio of 0.6 between the pumped flow (q_6) and the main flow (q_8) by controlling the pump speed. Another objective was to study the interactions between the pump speed feedback loop and the operation of the PRV. The overall aims can be summarised as follows:

- give a proof of concept - that the methodology works
- study the feasibility of maintaining the flow mixing ratio at the desired value
- study the dynamic interactions between the PRV operation and the pump speed control
- study the role of TCV1 in the dynamic performance of the system
- study the role of storage

Case Study Model

The model in Fig. 3 has altogether nine nodes, nine elements, one reservoir and one tank. The set of connection nodes is, $\mathbb{N}_c = \{n_1, n_2, n_3, n_4, n_5, n_6, n_7\}$. The set of connection nodes with pipes only is, $\mathbb{N}_{c,p} = \{n_4, n_5\}$. The set of connection nodes with control elements is, $\mathbb{N}_{c,ctr} = \{n_1, n_2, n_3, n_6, n_7\}$. The set of pipes is, $\mathbb{E}_p = \{e_1, e_4, e_5, e_6, e_8\}$. The set of control elements is, $\mathbb{E}_{ctr} = \{e_2, e_3, e_7, e_9\}$. The number of independent flows is, $e_{p,ind} = e - n_c = 2$ and the set of independent flows was selected as $\mathbb{E}_{p,ind} = \{e_1, e_6\}$. The node-element incidence matrix is displayed in Eq. 38

$$\mathbf{\Lambda}_c = \begin{bmatrix} +1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & +1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & +1 & +1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & -1 \end{bmatrix} \tag{38}$$

The loop-element incidence matrix is given by Eq. 39.

$$\Gamma = \begin{bmatrix} -1 & -1 & -1 & -1 & -1 & +1 & -1 & 0 & 0 \\ -1 & -1 & -1 & -1 & -1 & 0 & 0 & -1 & -1 \end{bmatrix} \tag{39}$$

A core section of the MATLAB code for the differential part is listed below. The names of the variables correspond to the names in the mathematical equations and to the names in Fig. 3 and should be easily identified.

```
%Pipes
q1_dot = -R1*q1*abs(q1)) + M1*(h0 - h1);
q6_dot = -R6*q6*abs(q6) + M6*(h6 - h5);

%PRV
if (hset - h2) ≥ 0
   xm_dot = aop*(hset - h2);
 else
  xm_dot = acl*(hset - h2);
 end;

%Pump
s_dot = -(1/T)*s + (1/T)*v;

%Tank
h8_dot = q9/At - d2/At;
```

For this case-study the algebraic part is more complicated than the differential part and will be described in plain text. First, it is necessary to prepare linear equations to calculate the vector of dependent flows, Eq. 29. Matrix $\mathbf{\Lambda}_{c,dep}$ is constructed by removing columns 1 and 6 from matrix $\mathbf{\Lambda}_c$. These columns correspond to the independent flows. Matrix $\mathbf{\Lambda}_{c,p,ind}$ on the right-hand side of the equation is made of columns 1 and 6 as displayed in Eq. 40

$$\begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ +1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & +1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +1 & -1 \end{bmatrix} \cdot \begin{bmatrix} q2 \\ q3 \\ q4 \\ q5 \\ q7 \\ q8 \\ q9 \end{bmatrix} = \begin{bmatrix} +1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & +1 \\ 0 & -1 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} q1 \\ q6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ d1 \\ 0 \\ 0 \\ d3 \end{bmatrix} \tag{40}$$

The case study system has a tree structure, and the dependent flows can be calculated explicitly one by one. This can be seen after executing multiplications on both sides of Eq. 40.

Let us consider a form of Eq. 30 and 31 for this case. TCV2 and the pump are connected on one side to a forced head node, TCV2 to the atmosphere and the pump to the tunnel. The equations corresponding to Eq. 30 are,

$$\begin{aligned} \Delta h_7 &= A(\frac{q_7}{u})^2 + B(\frac{q_7}{u})s + Cs^2 \\ \Delta h_9 &= K_{v2}\frac{1}{2gA_{v2}^2}q_9^2 \end{aligned} \tag{41}$$

PRV and TCV1 are connected on both ends to connection nodes. The equations corresponding to Eq. 31 are;

$$\begin{aligned} \Delta h_2 &= \frac{1}{C_v(x_m)^2}q_1^2 \\ \Delta h_3 &= K_{v1}\frac{1}{2gA_{v1}^2}q_1^2 \end{aligned} \tag{42}$$

Node 4 and node 5 are nodes with pipes only, so the equations corresponding to Eq. 32 are:

$$\begin{aligned} M_4\Delta h_4 - M_5\Delta h_5 &= R_4 q_4|q_4| - R_5 q_5|q_5| + \dot{d}_1 \\ M_5\Delta h_5 + M_6\Delta h_6 - M_8\Delta h_8 &= R_5 q_5|q_5| + R_6 q_6|q_6| - R_8 q_8|q_8|. \end{aligned} \tag{43}$$

The equation corresponding to the general form in Eq. 33 is Eq. 44. Nodes 1, 2 and 3 have been merged together to give a 'differential' mass balance equation.

$$M_1\Delta h_1 - M_4\Delta h_4 = R_1 q_1|q_1| - R_4 q_4|q_4| \tag{44}$$

Finally, Eqs 45 stem from the energy conservation law for the two pseudo-loops, one from the source h_0 to the tunnel and one from the source h_0 to the discharge node to the tank.

$$\begin{aligned} -\Delta h_1 - \Delta h_4 - \Delta h_5 + \Delta h_6 &= h_{tunnel} - h_0 + \Delta h_2 + \Delta h_3 + \Delta h_7 \\ -\Delta h_1 - \Delta h_4 - \Delta h_5 - \Delta h_8 &= h_{discharge} - h_0 + \Delta h_2 + \Delta h_3 + \Delta h_9 \end{aligned} \tag{45}$$

Eqs 43, 44 and 45 form a system of linear algebraic equations with unknown variables $\Delta h_1, \Delta h_4, \Delta h_5, \Delta h_6, \Delta h_8$ as displayed in Eq. 46.

$$\begin{bmatrix} 0 & M_4 & -M_5 & 0 & 0 \\ 0 & 0 & M_5 & M_6 & -M_8 \\ M_1 & -M_4 & 0 & 0 & 0 \\ -1 & -1 & -1 & +1 & 0 \\ -1 & -1 & -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} \Delta h_1 \\ \Delta h_4 \\ \Delta h_5 \\ \Delta h_6 \\ \Delta h_8 \end{bmatrix} = \begin{bmatrix} R_4 q_4|q_4| - R_5 q_5|q_5| + \dot{d}_1 \\ R_5 q_5|q_5| + R_6 q_6|q_6| - R_8 q_8|q_8| \\ R_1 q_1|q_1| - R_4 q_4|q_4| \\ h_{tunnel} - h_0 + \Delta h_2 + \Delta h_3 + \Delta h_7 \\ h_{discharge} - h_0 + \Delta h_2 + \Delta h_3 + \Delta h_9 \end{bmatrix} \tag{46}$$

The reduced node-element incidence matrix is given by Eq. 47.

$$\mathbf{\Lambda}_{c,red} = \begin{bmatrix} 0 & M_4 & -M_5 & 0 & 0 \\ 0 & 0 & M_5 & M_6 & -M_8 \\ M_1 & -M_4 & 0 & 0 & 0 \end{bmatrix} \tag{47}$$

Numerical Results

The fundamental task was to maintain a constant ratio of 0.6 between the pumped flow q_6 and the main flow q_8 by manipulating the pump speed. The pump flow and the main flow are output variables from the WDS model as depiceted in Figure 4.

They are averaged over 12-minute intervals and the ratio of the averages is calculated. The ratio signal and actual speed signal s are inputs to the speed controller. The controller calculates a new set-point for the pump speed using a simple algorithm listed below.

```
function v = speed_rule(ratio,s)
 if ratio < 0.58
  v = s + 0.002;
 elseif ratio > 0.62
   v = s - 0.002;
 else
   v = s;
 end
```

If the ratio drops below 0.58, the speed set-point equals $(s + 0.002)$. If the ratio is above 0.62 the speed set-point equals $(s - 0.002)$. Otherwise, it is set to the value of the current speed s. The speed set-point is updated every 10 minutes. A too long sampling time may lead to poor control performance. The second control loop is to manage the water level in the tank. The storage control allows to equalise the main flow and also the pump flow which is an important operational requirement related to the tunnel source. The graphical presentation of the rule is depicted in Fig. 5.

The tank head is on the x-axis and varies from $180m$ to $184m$. The y-axis shows the K_{v2} value of TCV2, which controls the inflow to the tank. It varies from 800 to 4000 for the considered valve diameter of $0.2m$. K_{v2} depends on the tank level and time. The continuous line depicts the night operation and the dashed line the day operation. During the night,K_{v2} is low to allow a big inflow to fill the tank. When the tank is 75% full, the minor loss coefficient increases linearly to prevent overflowing. The day operation is depicted by the dashed line. Normally K_{v2} is high (4000) to allow a gradual emptying of the tank by the day demands. However, if the tank level drops below 25% of its capacity, the minor loss coefficient decreases linearly to prevent emptying of the tank

Changes to K_{v2} are applied via the rate limiter block in Simulink to prevent abrupt changes and to avoid unwanted transients in the water system. The three demand signals over a seven-day period are displayed in Fig. 6.

Demand d_1 is an export of water to another WDS and is constant. Demand d_3 represents the flow to the downstream system and constitutes a boundary condition to the case-study model. It is characterized by rapid changes up and down resulting from pump operation in the downstream part of the system. Finally, demand d_2 from the tank covers a local urban area and has a typical pattern of morning and evening peaks. It was expected that TCV1 can have a stabilising effect on the behaviour of the WDS control system by introducing an additional head loss in the main pipe between the PRV and the flow mixing point, h_5. The minor loss coefficient K_{v1} equals 30 for the valve diameter $0.2m$ which produces a head loss of approximately $10m$ for a range of flows in the WDS. The PRV head set-point is $264m$ to provide sufficient pressure at a critical point in the downstream system. The parameters of the pipes and control components are included in Table 1 and Table 2, respectively. The simulation run over seven days ($168\ hours$) on a desktop PC with an i5 core Intel processor. The average elapse time was approximately one minute. The tank head signal is displayed in Fig. 7.

The time starts at midnight. The tank fills during the night and empties during the day covering local demands d_2 as shown in Fig. 6. The head varies within the feasible range between $180m$ and $184m$. The major aim was to verify whether the flow mixing ratio, $\frac{q_6}{q_8}$, is maintained at the desired level of 0.6. The corresponding signal is depicted in Fig. 8.

The ratio is controlled very well around the desired value of 0.6. Deviations from this value are caused by sudden changes in demand d_3 in the downstream part of the system. The PRV worked well, maintaining the set-point at $264m$. The outlet head together with the valve opening signal are shown in Fig. 9.

Subsequently, TCV1 was removed to check the behaviour of the WDS control system without the valve. The two ratio signals with and without TCV1 are shown in Fig. 10. When TCV1 was not present, the PRV set-point was changed from $264m$ to $254m$ to maintain the same boundary head h_7 for the downstream system.

After removing TCV1, the quality of the control deteriorated with the ratio going up to 0.73 and down to 0.47. In contrast, the variation was within the interval [0.57 0.63] when the valve was present. A similar comparison was done for PRV opening and is displayed in Fig. 11.

Compared to the experiment when TCV1 was present, the average value of the PRV opening was lower to maintain a bigger head drop across the valve and the variation of the opening was much bigger.

The corresponding plots of the PRV outlet head, h_2, are displayed in Fig. 12.

The two signals look very similar. The sudden blips correspond to rapid changes in the main demand d_3 caused by pump switching in the downstream water system. Each time the outlet head returned quickly to the desired set-point value of $264m$ for the system with TCV1 and $254m$ for the system without the valve. The standard deviation of the signals was 0.117 and 0.17 when TCV1 was removed. Variations in the PRV outlet head had a greater impact on the main flow and subsequently on the mixing ratio when TCV1 was absent as shown in Fig. 10.

The pump speed set-point v was updated every ten minutes. Increasing this sampling time up to one hour did not have a significant effect on the quality of the control. The influence of the sampling time was observed only in the situation of low flow in the main pipe when the demands d_2 and d_3 were reduced by half and TCV1 was not present. The signals for sampling time equal to 10 min, 20 min and 40 min over one day are depicted in Fig. 13.

CONCLUSIONS

The paper presented a methodology for modelling the dynamic behaviour of water distribution systems equipped with control elements and control loops. Pipes were represented by a rigid column model, storage tanks by standard storage equations and control elements with local control loops like PRVs or elements with significant inertia like pumps by ordinary differential equations. Other components such as TCV or leakage were represented by algebraic equations. The overall dynamic WDS model is an example of a differential algebraic equation system with index 1. The algebraic part has the form of a system of simultaneous linear equations with head loss on elements as unknown variables. The nodal heads can be calculated from an additional linear algebraic equation. The model was implemented in the MATLAB/Simulink environment which facilitates such an implementation by providing a simulation engine and a wide collection of solvers and user-defined functions. The implementation is based on a differential part and an algebraic part. The differential part calculates the right-hand side of the differential equations and the algebraic part solves the simultaneous linear equations using the'linsolve' MATLAB function. MATLAB also supports the implementation of advanced user-defined control algorithms. The general methodology was applied to an industrial case-study on a regional water supply system. The aim was to investigate the possibility of maintaining a constant flow mixing ratio at the pipe junction by controlling the pump speed. The study also considered the dynamic interactions between the pump speed and the PRV operation in a very low loss system. It appeared that after placing the TCV1 valve on the main pipe, it was possible to maintain the flow mixing ratio at the desired level of 0.6. However, when TCV1 was removed, the disturbance rejection was poor and the ratio varied widely between 0.47 and 0.73. The engineering explanation for this is that in a low loss system, small changes of the outlet head from PRV caused significant changes in the main flow and subsequently significant changes in the mixing ratio. In more precise terms, the system curve, flow against head as seen from the pump was very flat and after introducing TCV1 the slope of the system curve has increased improving stability of the system. An important stabilising role was also played by the tank, which acted as a buffer between the main flow and the variable demands from the tank, allowing to equalise the main flow between day and night and avoiding low night flows. Experimental results for low demand and almost constant tank level indicated poor disturbance rejection performance. The sampling period of the pump speed control loop did not have a significant impact on the system behaviour in normal conditions (with TCV1 and tank). However, for low flows (demands), increase in the sampling time caused significant deterioration in control performance.

DATA AVAILABILITY STATEMENT

The following data, models, or code generated or used during the study are available from the corresponding author by request:

- Simulink model of the case-study
- MATLAB script which initialise the data for the Simulink model
- MATLAB workspace which contains time-varying demand

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NOTATION

The following symbols are used in this paper:

A_l	=	cross-section area of leakage;
A_t	=	cross-section area of a tank;
A, B, C	=	coefficients of the hydraulic pump characteristic;
$A_{cv},\ B_{cv},\ C_{cv}$	=	coefficients of the PRV capacity characteristic;
$C_v(x_m)$	=	PRV capacity characteristic;
$\mathbb{E}$	=	set of all elements in a WDS;
$\mathbb{E}_{c,dyn}$	=	set of dynamic control elements;
$\mathbb{E}_{c,stat}$	=	set of static control elements;
$\mathbb{E}_{ctr}$	=	set of all control elements;
$\mathbb{E}_{ctr,c}$	=	set of control elements connected between two connection nodes;
$\mathbb{E}_{ctr,f}$	=	set of control elements connected between a forced head node and a connection node;
$\mathbb{E}_i$	=	set of elements connected to node i;
$\mathbb{E}_{i_{d,j}}$	=	set of pipes connected to the destination node of the control element j;
$\mathbb{E}_{i_{o,j}}$	=	set of pipes connected to the destination node of the control element j;
$\mathbb{E}_p$	=	set of all pipes;
$\mathbb{E}_{p,ind}$	=	set of pipes with independent flows;

K_v = minor loss coefficient for the TCV valve;

M_j = coefficient in the pipe equation for pipe j;

$\mathbf{M}$ = $(e_p \times e_p)$ diagonal matrix with M_j entries on the diagonal;

$\mathbb{N}$ = set of all nodes in WDS;

$\mathbb{N}_c$ = set of connection nodes;

$\mathbb{N}_{c,p}$ = set of connection nodes with pipes only;

$\mathbb{N}_{c,ctr}$ = set of connection nodes with pipes and control element;

$\mathbb{N}_f$ = set of nodes with forced head;

R_j = coefficient in the pipe equation for pipe j;

$\mathbf{R}(\mathbf{q})$ = column vector with $R_j q_j |q_j|$ elements;

T = time constant corresponding to pump inertia;

a = dynamic variable of a generic dynamic control element;

$\mathbf{a}$ = vector of dynamic variables of dynamic control elements;

c = control variable of a generic control element;

$\mathbf{d}_c$ = vector of demands at connection nodes;

$\mathbf{d}_{c,red}$ = vector of demands at connection nodes in a reduced model;

e = number of elements;

e_p = number of pipes in the model;

e_{ctr} = number of control elements;

h_d = head at destination node of an element;

h_o = head at origin node of an element;

h_set = head set-point of a PRV;

h_t = head at a tank;

$\mathbf{h}_t$ = vector of tank heads;

n = number of nodes in the model;

n_c = number of connection nodes in the model;

n_f = number of nodes with forced head;

$\mathbf{q}$ = e vector of flows in all elements;

$\mathbf{q}_{dep}$ = vector of dependent flows in element;

$\mathbf{q}_{p,ind}$ = vector of independent flows in pipes;

q_j = flow in an element j;

s = relative pump speed;

x_m = PRV opening in %;

$\Delta\mathbf{h}$ = vector of head losses along all elements;

$\Delta\mathbf{h}_{f-1}$ = vector of head differences between the reference node and other forced head nodes;

$\Delta\mathbf{h}_p$ = vector of head losses along pipes;

$\mathbf{\Gamma}$ = loop-element incidence matrix;

$\mathbf{\Gamma}_l$ = loop-element incidence matrix for fundamental loops;

$\mathbf{\Gamma}_{f-1}$ = loop-element incidence matrix for pseudo-loops;

$\mathbf{\Gamma}_p$ = loop-pipes incidence matrix;

$\mathbf{\Gamma}_{ctr}$ = loop- control elements incidence matrix;

$\mathbf{\Lambda}$ = node-element incidence matrix;

$\mathbf{\Lambda}_c$ = node-element incidence matrix for connection nodes;

$\mathbf{\Lambda}_{c,dep}$ = incidence matrix for connection nodes and dependent flows;

$\mathbf{\Lambda}_{c,p,ind}$ = incidence matrix for connection nodes and pipe independent flows;

$\mathbf{\Lambda}_f$ = ode-element incidence matrix for forced head nodes;

$\mathbf{\Lambda}_{c,red}$ = node-element incidence matrix for connection nodes for reduced model;

α_{open} = rate of opening of a PRV; and

α_{close} = rate of closing of a PRV.

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List of Tables

TABLE 1. Pipe data

Id	Diameter [m]	Length [m]	DW friction factor
1	0.8	46212	0.015
4	0.8	9750	0.015
5	0.8	10	0.015
6	0.8	10	0.015
8	0.8	21179	0.015

TABLE 2. Control elements data

Id	Name	Equation	parameter values
2	PRV	Eq. 3, and Eq. 4	$\alpha_{open} = 0.005,\ \alpha_{closed} = 0.03$ $A_{cv} = 4.9472e-06,\ B_{cv} = 4.9472e-06,\ C_{cv} = -0.001$
3	TCV1	Eq. 2	$D = 0.2,\ K_v = 30$
9	TCV2	Eq. 2	$D = 0.2,\ K_v$ variable
7	Pump	Eq. 5 and Eq. 6	$T = 20sec,\ A = -4.7609e+03\ B = 0,\ C = 200$

List of Figures

Fig. 1. Structure of the Water Distribution System dynamic model

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Fig. 4. Case study system with controllers

Fig. 5. TCV2 rule for tank storage control

Fig. 6. Demands in the system

Fig. 7. Tank head [m]

Fig. 8. Flow mixing ratio with TCV1 present

Fig. 9. PRV outlet head and valve opening

Fig. 10. Flow mixing ratio with and without TCV1

Fig. 11. PRV opening with and without TCV1

Fig. 12. PRV outlet head h2 with and without TCV1

Fig. 13. Effect of sampling on ratio signal at low demands